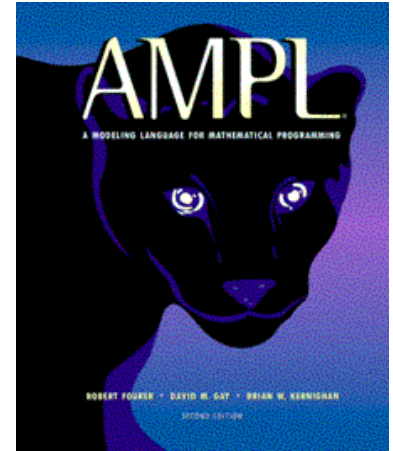


# Attacking Hard Mixed-Integer Optimization Problems through the AMPL Modeling Language



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# AMPL: Work Scheduling Example

## *Cover demands for workers*

- ❖ Each “shift” requires a certain number of employees
- ❖ Each employee works a certain “schedule” of shifts
- ❖ *Each schedule that is worked by anyone must be worked by a fixed minimum number*

## *Minimize total workers needed*

- ❖ Which schedules are used?
- ❖ How many work each of schedule?

# AMPL

## *Algebraic modeling language: symbolic data*

```
set SHIFTS;                # shifts
param Nsched;              # number of schedules;
set SCHEDS = 1..Nsched;   # set of schedules

set SHIFT_LIST {SCHEDS} within SHIFTS;

param rate {SCHEDS} >= 0;    # pay rates
param required {SHIFTS} >= 0; # staffing requirements
param least_assign >= 0;     # min workers on any schedule used
```

# AMPL

## *Algebraic modeling language: symbolic model*

```
var Work {SCHEDS} >= 0 integer;
var Use  {SCHEDS} >= 0 binary;

minimize Total_Cost:
    sum {j in SCHEDS} rate[j] * Work[j];

subject to Shift_Needs {i in SHIFTS}:
    sum {j in SCHEDS: i in SHIFT_LIST[j]} Work[j] >= required[i];

subject to Least_Use1 {j in SCHEDS}:
    least_assign * Use[j] <= Work[j];

subject to Least_Use2 {j in SCHEDS}:
    Work[j] <= (max {i in SHIFT_LIST[j]} required[i]) * Use[j];
```

# AMPL

## *Explicit data independent of symbolic model*

```
set SHIFTS := Mon1 Tue1 Wed1 Thu1 Fri1 Sat1
            Mon2 Tue2 Wed2 Thu2 Fri2 Sat2
            Mon3 Tue3 Wed3 Thu3 Fri3 ;

param Nsched := 126 ;

set SHIFT_LIST[1] := Mon1 Tue1 Wed1 Thu1 Fri1 ;
set SHIFT_LIST[2] := Mon1 Tue1 Wed1 Thu1 Fri2 ;
set SHIFT_LIST[3] := Mon1 Tue1 Wed1 Thu1 Fri3 ;
set SHIFT_LIST[4] := Mon1 Tue1 Wed1 Thu1 Sat1 ;
set SHIFT_LIST[5] := Mon1 Tue1 Wed1 Thu1 Sat2 ;      .....

param required := Mon1 100  Mon2 78  Mon3 52
                  Tue1 100  Tue2 78  Tue3 52
                  Wed1 100  Wed2 78  Wed3 52
                  Thu1 100  Thu2 78  Thu3 52
                  Fri1 100  Fri2 78  Fri3 52
                  Sat1 100  Sat2 78 ;
```

# AMPL

## *Solver independent of model & data*

```
ampl: model sched1.mod;
ampl: data sched.dat;

ampl: let least_assign := 7;

ampl: option solver cplex;
ampl: solve;

CPLEX 12.1.0: optimal integer solution; objective 266
473 MIP simplex iterations
72 branch-and-bound nodes

ampl: option omit_zero_rows 1, display_1col 0;
ampl: display Work;

Work [*] :=
  2 12    16 14    29  7    53  7    91 17    112  9    122 29
  3  7    18  7    37 21    78 21    100 19    116 20    124  7
  6 10    20  7    41  8    82 21    109  7    118 16

;
```

# AMPL

## *Language independent of solver*

```
ampl: option solver gurobi;
```

```
ampl: solve;
```

```
Gurobi 3.0.0: optimal solution; objective 266
```

```
396 simplex iterations
```

```
3 branch-and-cut nodes
```

```
ampl: display Work;
```

```
Work [*] :=
```

```
  1 29    37 35    84 18    91 17    101 11    112 18    118 17  
 21 36    71  7    89 18    95  7    109 10    116  7    124 36
```

```
;
```

# Topics

*1: Look at the details*

*2: Know when to quit*

*3: Multiprocess*

*4: Tune*

*5: Reformulate*

*6: “Cheat” on the method*

*7: “Cheat” on the data*

# 1: Look at the Details

*Log lines*

*Preprocessing*

*Postprocessing*

# Log Lines

## *CPLEX*

```
ampl: option cplex_options 'mipdisplay 2 mipinterval 100';
```

```
ampl: solve;
```

```
CPLEX 11.2.0: mipdisplay 2
```

```
mipinterval 100
```

	Nodes					Cuts/		
	Node	Left	Objective	IInf	Best Integer	Best Node	ItCnt	Gap
	0	0	265.6000	18		265.6000	38	
	0	0	265.6000	22		Cuts: 12	46	
	0	0	265.6000	21		MIRcuts: 1	54	
*	0+	0			10348.0000	265.6000	54	97.43%
	0	2	265.6000	18	10348.0000	265.6000	54	97.43%
	100	102	267.0000	6	10348.0000	265.6000	537	97.43%
*	119	118	integral	0	273.0000	265.6000	882	2.71%
	200	199	265.7500	19	273.0000	265.6000	1633	2.71%
*	244	215	integral	0	268.0000	265.6000	2744	0.90%
	300	271	266.5000	14	268.0000	265.6000	3093	0.90%
*	344+	1			266.0000	265.6000	3323	0.15%

# Log Lines *(cont'd)*

## Work

	Nodes					Cuts/		
	Node	Left	Objective	IInf	Best Integer	Best Node	ItCnt	Gap
	0	0	265.6000	18		265.6000	38	
	0	0	265.6000	22		Cuts: 12	46	
	0	0	265.6000	21		MIRcuts: 1	54	
*	0+	0			10348.0000	265.6000	54	97.43%
	0	2	265.6000	18	10348.0000	265.6000	54	97.43%
	100	102	267.0000	6	10348.0000	265.6000	537	97.43%
*	119	118	integral	0	273.0000	265.6000	882	2.71%
	200	199	265.7500	19	273.0000	265.6000	1633	2.71%
*	244	215	integral	0	268.0000	265.6000	2744	0.90%
	300	271	266.5000	14	268.0000	265.6000	3093	0.90%
*	344+	1			266.0000	265.6000	3323	0.15%

# Log Lines *(cont'd)*

## *Lower bounds*

	Nodes					Cuts/		
	Node	Left	Objective	IInf	Best Integer	Best Node	ItCnt	Gap
	0	0	265.6000	18		265.6000	38	
	0	0	265.6000	22		Cuts: 12	46	
	0	0	265.6000	21		MIRcuts: 1	54	
*	0+	0			10348.0000	265.6000	54	97.43%
	0	2	265.6000	18	10348.0000	265.6000	54	97.43%
	100	102	267.0000	6	10348.0000	265.6000	537	97.43%
*	119	118	integral	0	273.0000	265.6000	882	2.71%
	200	199	265.7500	19	273.0000	265.6000	1633	2.71%
*	244	215	integral	0	268.0000	265.6000	2744	0.90%
	300	271	266.5000	14	268.0000	265.6000	3093	0.90%
*	344+	1			266.0000	265.6000	3323	0.15%

# Log Lines *(cont'd)*

## *Upper bounds*

	Nodes		Objective	IInf	Best Integer	Cuts/		Gap
Node	Left	Best Node				ItCnt		
	0	0	265.6000	18		265.6000	38	
	0	0	265.6000	22		Cuts: 12	46	
	0	0	265.6000	21		MIRcuts: 1	54	
*	0+	0			10348.0000	265.6000	54	97.43%
	0	2	265.6000	18	10348.0000	265.6000	54	97.43%
	100	102	267.0000	6	10348.0000	265.6000	537	97.43%
*	119	118	integral	0	273.0000	265.6000	882	2.71%
	200	199	265.7500	19	273.0000	265.6000	1633	2.71%
*	244	215	integral	0	268.0000	265.6000	2744	0.90%
	300	271	266.5000	14	268.0000	265.6000	3093	0.90%
*	344+	1			266.0000	265.6000	3323	0.15%

# Log Lines *(cont'd)*

## Gap

	Nodes					Cuts/		
	Node	Left	Objective	IInf	Best Integer	Best Node	ItCnt	Gap
	0	0	265.6000	18		265.6000	38	
	0	0	265.6000	22		Cuts: 12	46	
	0	0	265.6000	21		MIRcuts: 1	54	
*	0+	0			10348.0000	265.6000	54	97.43%
	0	2	265.6000	18	10348.0000	265.6000	54	97.43%
	100	102	267.0000	6	10348.0000	265.6000	537	97.43%
*	119	118	integral	0	273.0000	265.6000	882	2.71%
	200	199	265.7500	19	273.0000	265.6000	1633	2.71%
*	244	215	integral	0	268.0000	265.6000	2744	0.90%
	300	271	266.5000	14	268.0000	265.6000	3093	0.90%
*	344+	1			266.0000	265.6000	3323	0.15%

# Preprocessing

## *Gurobi*

```
ampl: option gurobi_options 'outlev 1 logfreq 1 timing 1';  
ampl: option presolve 10;  
ampl: option show_stats 1;
```

```
ampl: solve;
```

**Presolve** eliminates 159622 constraints and 755655 variables.

Adjusted problem:

**385720 variables:**

384720 binary variables

1000 linear variables

**317322 constraints**, all linear; 13961712 nonzeros

1 linear objective; 308776 nonzeros.

Gurobi 3.0.0:

Optimize a model with **317322 Rows**, **385720 Columns** and 13961712 NonZeros

**Presolve** removed 8335 rows and 41141 columns (presolve time = 2s) ...

**Presolve** removed 11605 rows and 46958 columns (presolve time = 12s) ...

**Presolve** removed 11605 rows and 52750 columns (presolve time = 12s) ...

**Presolve** removed 11605 rows and 56770 columns (presolve time = 13s) ...

**Presolve** removed 12944 rows and 58110 columns (presolve time = 16s) ...

# Preprocessing (*cont'd*)

## *Gurobi*

```
Presolve removed 218544 rows and 58411 columns (presolve time = 16s) ...
Presolve removed 218544 rows and 58411 columns (presolve time = 18s) ...
Presolve removed 218544 rows and 58411 columns (presolve time = 22s) ...
Presolve removed 218544 rows and 58411 columns (presolve time = 27s) ...
Presolve removed 219031 rows and 58411 columns (presolve time = 31s) ...
Presolve removed 222957 rows and 189567 columns (presolve time = 31s) ...
Presolve removed 264766 rows and 292409 columns (presolve time = 32s) ...
Presolve removed 270648 rows and 318250 columns (presolve time = 33s) ...
Presolve removed 285755 rows and 318284 columns (presolve time = 35s) ...
Presolve removed 285781 rows and 318284 columns (presolve time = 36s) ...
Presolve removed 289794 rows and 342963 columns (presolve time = 37s) ...
Presolve removed 297321 rows and 346355 columns (presolve time = 38s) ...
Presolve removed 300053 rows and 355682 columns (presolve time = 39s) ...
Presolve removed 300815 rows and 357734 columns (presolve time = 40s) ...
Presolve removed 301076 rows and 358221 columns (presolve time = 41s) ...
Presolve removed 301276 rows and 358621 columns
```

```
Presolve time: 42.87s
```

```
Presolved: 16046 Rows, 27099 Columns, 281126 Nonzeros
```

# Preprocessing *(cont'd)*

## *Gurobi*

Objective GCD is 0.001

Found heuristic solution: objective 136.2260000

Found heuristic solution: objective 29.8440000

Root relaxation: objective 1.592000e+00, 7204 iterations, 0.12 seconds

Nodes		Current Node		Objective Bounds			Work		
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1.5920000	1.59200	0.0%	-	43s

Explored 0 nodes (7204 simplex iterations) in 43.31 seconds

Thread count was 8 (of 8 available processors)

Optimal solution found (tolerance 1.00e-04)

Best objective 1.5920000000e+00, best bound 1.5920000000e+00, gap 0.0%

# Postprocessing

## *Gurobi*

Explored 10161 nodes (609669 simplex iterations) in 146.98 seconds

Thread count was 8 (of 8 available processors)

Node limit reached

Best objective 1.200000000e+01, best bound 8.00000000e+00, gap 33.3333%

Optimize a model with 4112 Rows, 2439 Columns and 15791 NonZeros

Iteration	Objective	Primal Inf.	Dual Inf.	Time
0	0.0000000e+00	2.350000e+02	0.000000e+00	0s
35	1.2000000e+01	0.000000e+00	0.000000e+00	0s

Solved in 35 iterations and 0.00 seconds

Optimal objective 1.200000000e+01

Gurobi 3.0.0: node limit; objective 12

609669 simplex iterations

10161 branch-and-cut nodes

plus 35 simplex iterations for intbasis

## 2: Know When to Quit

### *Difficult situations*

- ❖ Work scheduling without log lines
- ❖ Work scheduling with log lines
- ❖ Balanced assignment: a complete failure

### *Can we stop early?*

- ❖ In these cases: good bet
- ❖ In general: not so clear

# Work Scheduling Without Log Lines

## *A day of CPLEX output*

```
ampl: model sched1.mod;
ampl: data sched.dat;

ampl: let least_assign := 19;

ampl: option solver cplex;
ampl: option cplex_options 'branch 1';
ampl: solve;

252 variables:
    126 binary variables
    126 integer variables
269 constraints, all linear; 1134 nonzeros
1 linear objective; 126 nonzeros.

CPLEX 12.1.0: branch 1
```

# Work Scheduling With Log Lines

*CPLEX starts promisingly . . .*

	Nodes				Best	Cuts/		
	Node	Left	Objective	IInf	Integer	Best Node	ItCnt	Gap
*	0+	0			12112.0000		40	---
	0	0	265.6000	25	12112.0000	265.6000	40	97.81%
	0	0	265.6000	29	12112.0000	Cuts: 12	79	97.81%
	0	0	265.6000	24	12112.0000	MIRcuts: 4	87	97.81%
	0	0	265.6000	24	12112.0000	MIRcuts: 3	123	97.81%
	0	2	265.6000	24	12112.0000	265.6000	123	97.81%
*	12+	12			616.0000	265.6000	198	56.88%
*	12+	12			277.0000	265.6000	198	4.12%
*	560+	83			276.0000	265.6000	4897	3.77%
*	566+	54			269.0000	265.6000	5117	1.26%
	10000	2132	268.0000	9	269.0000	265.6000	157659	1.26%
	20000	4057	cutoff		269.0000	265.6000	327850	1.26%
	30000	5926	266.3333	12	269.0000	265.6000	507164	1.26%
	40000	7647	cutoff		269.0000	265.6000	690189	1.26%
	50000	9517	268.0000	23	269.0000	265.6000	884164	1.26%
	60000	11228	265.7500	25	269.0000	265.6000	1088023	1.26%
	70000	13361	268.0000	17	269.0000	265.6000	1276209	1.26%
	80000	15594	268.0000	17	269.0000	265.6000	1462445	1.26%

# Work Scheduling With Log Lines *(cont'd)*

*... tightens the bound ...*

Node	Nodes Left	Objective	IInf	Best Integer	Cuts/ Best Node	ItCnt	Gap
1160000	196770	268.0000	15	269.0000	266.2500	24998690	1.02%
1170000	197681	268.0000	6	269.0000	266.3333	25285264	0.99%
1180000	198093	268.0000	6	269.0000	266.5000	25567868	0.93%
1190000	198138	266.6667	24	269.0000	266.6667	25804095	0.87%
1200000	198604	267.2857	17	269.0000	266.6667	26063901	0.87%
Elapsed real time = 263.20 sec. (tree size = 66.83 MB)							
1210000	198978	cutoff		269.0000	267.0000	26306006	0.74%
1220000	198908	267.0000	9	269.0000	267.0000	26673281	0.74%
1230000	199278	cutoff		269.0000	267.0000	27003874	0.74%
1240000	199565	268.0000	10	269.0000	267.2500	27261518	0.65%
1250000	200080	268.0000	8	269.0000	267.3333	27521492	0.62%
1260000	200971	cutoff		269.0000	267.3333	27789325	0.62%
1270000	201601	cutoff		269.0000	267.5000	28036060	0.56%
1280000	202092	cutoff		269.0000	267.7500	28247750	0.46%
1290000	202643	268.0000	4	269.0000	268.0000	28449503	0.37%
1300000	203307	cutoff		269.0000	268.0000	28648240	0.37%
Elapsed real time = 286.48 sec. (tree size = 68.31 MB)							
1310000	203837	cutoff		269.0000	268.0000	28850298	0.37%
1320000	203880	cutoff		269.0000	268.0000	29058072	0.37%

# Work Scheduling With Log Lines *(cont'd)*

*... eventually stops expanding the search tree ...*

Node	Nodes Left	Objective	IInf	Best Integer	Cuts/ Best Node	ItCnt	Gap
387790000	550587	cutoff		269.0000	268.0000	1.40870e+10	0.37%
387800000	550593	268.0000	10	269.0000	268.0000	1.40874e+10	0.37%
Elapsed real time = 95728.25 sec. (tree size = 155.89 MB)							
Nodfile size = 28.64 MB (19.83 MB after compression)							
387810000	550586	cutoff		269.0000	268.0000	1.40879e+10	0.37%
387820000	550532	cutoff		269.0000	268.0000	1.40884e+10	0.37%
387830000	550535	cutoff		269.0000	268.0000	1.40888e+10	0.37%
387840000	550540	cutoff		269.0000	268.0000	1.40893e+10	0.37%
387850000	550544	268.0000	18	269.0000	268.0000	1.40898e+10	0.37%
387860000	550557	268.0000	13	269.0000	268.0000	1.40902e+10	0.37%
387870000	550578	268.0000	7	269.0000	268.0000	1.40907e+10	0.37%
387880000	<b>550611</b>	268.0000	17	269.0000	268.0000	1.40911e+10	0.37%
387890000	550564	268.0000	6	269.0000	268.0000	1.40916e+10	0.37%
387900000	550529	268.0000	5	269.0000	268.0000	1.40920e+10	0.37%
Elapsed real time = 95766.47 sec. (tree size = 155.87 MB)							
Nodfile size = 28.64 MB (19.83 MB after compression)							
387910000	550448	268.0000	10	269.0000	268.0000	1.40923e+10	0.37%
387920000	550458	cutoff		269.0000	268.0000	1.40928e+10	0.37%
387930000	550436	cutoff		269.0000	268.0000	1.40932e+10	0.37%
387940000	550390	268.0000	7	269.0000	268.0000	1.40936e+10	0.37%
387950000	550334	268.0000	7	269.0000	268.0000	1.40940e+10	0.37%

# Work Scheduling With Log Lines *(cont'd)*

*... then takes “forever” to prove optimality*

Node	Nodes Left	Objective	IInf	Best Integer	Cuts/ Best Node	ItCnt	Gap
465550000	301	268.0000	6	269.0000	268.0000	1.72059e+10	0.37%
465560000	249	268.0000	12	269.0000	268.0000	1.72063e+10	0.37%
465570000	228	cutoff		269.0000	268.0000	1.72068e+10	0.37%
465580000	100	cutoff		269.0000	268.0000	1.72072e+10	0.37%
465590000	80	268.0000	6	269.0000	268.0000	1.72076e+10	0.37%

Flow cuts applied: 1  
Gomory fractional cuts applied: 1  
Root node processing (before b&c):  
Real time = 0.03  
Parallel b&c, 8 threads:  
Real time = 112012.60  
Sync time (average) = 23098.41  
Wait time (average) = 64021.17  
-----  
Total (root+branch&cut) = **112012.63 sec.**  
CPLEX 12.1.0: **optimal integer solution; objective 269**  
-2147483648 MIP simplex iterations  
**465596558 branch-and-bound nodes**

# Work Scheduling With Log Lines

*Gurobi starts promisingly . . .*

Found heuristic solution: objective 408.000000

Root relaxation: objective 2.656000e+02, 97 iterations, 0.00 seconds

Nodes		Current Node			Objective Bounds			Work		
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time	
	0	0	265.60000	0	15	408.00000	265.60000	34.9%	-	0s
H	0	0				290.00000	265.60000	8.41%	-	0s
	0	0	266.00000	0	24	290.00000	266.00000	8.28%	-	0s
H	0	0				275.00000	266.00000	3.27%	-	0s
	0	0	266.00000	0	21	275.00000	266.00000	3.27%	-	0s
	0	0	266.00000	0	21	275.00000	266.00000	3.27%	-	0s
	0	0	266.00000	0	12	275.00000	266.00000	3.27%	-	0s
	0	2	266.00000	0	7	275.00000	266.00000	3.27%	-	0s
H	42	50				269.00000	266.00000	1.12%	8.6	0s
	26094	12393	268.00000	65	19	269.00000	266.00000	1.12%	6.7	3s
	61393	28900	cutoff	51		269.00000	266.00000	1.12%	6.6	6s
	103339	48360	268.00000	43	7	269.00000	266.00000	1.12%	6.6	9s
	139466	64591	268.00000	57	7	269.00000	266.00000	1.12%	6.6	12s
	176702	80630	cutoff	71		269.00000	266.00000	1.12%	6.6	15s
	217155	97936	268.00000	29	17	269.00000	266.00000	1.12%	6.6	18s

# Work Scheduling With Log Lines *(cont'd)*

*... tightens the bound ...*

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
508581	228275	267.20000	66	26	269.00000	266.00000	1.12%	6.4	39s
555160	247970	268.00000	60	13	269.00000	266.00000	1.12%	6.4	42s
593775	263820	268.00000	80	15	269.00000	266.00000	1.12%	6.4	45s
637257	280342	268.00000	65	7	269.00000	266.00000	1.12%	6.4	48s
679843	295729	268.00000	55	14	269.00000	266.00000	1.12%	6.4	51s
724464	315739	268.00000	72	6	269.00000	266.00000	1.12%	6.4	54s
764318	331586	cutoff	68		269.00000	266.00000	1.12%	6.4	57s
807548	348225	268.00000	43	22	269.00000	266.00000	1.12%	6.3	60s
850968	362477	cutoff	56		269.00000	266.00000	1.12%	6.3	63s
908917	374846	268.00000	53	18	269.00000	266.75000	0.84%	6.2	66s
981286	392065	268.00000	57	9	269.00000	268.00000	0.37%	6.1	69s
1052711	400737	cutoff	58		269.00000	268.00000	0.37%	6.2	72s
1121014	403413	cutoff	56		269.00000	268.00000	0.37%	6.2	75s
1194809	403685	268.00000	57	17	269.00000	268.00000	0.37%	6.3	78s
1271977	405824	268.00000	85	7	269.00000	268.00000	0.37%	6.3	81s
1321885	408302	cutoff	88		269.00000	268.00000	0.37%	6.3	84s
1392557	411299	268.00000	62	7	269.00000	268.00000	0.37%	6.3	87s
1462331	410872	cutoff	58		269.00000	268.00000	0.37%	6.4	90s
1525446	412952	268.00000	64	7	269.00000	268.00000	0.37%	6.4	93s

# Work Scheduling With Log Lines *(cont'd)*

*... eventually stops expanding the search tree ...*

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
468100473	4333549	268.00000	63	6	269.00000	268.00000	0.37%	7.6	23514s
468158488	4333932	cutoff	55		269.00000	268.00000	0.37%	7.6	23517s
468217704	4334267	cutoff	51		269.00000	268.00000	0.37%	7.6	23520s
468270818	4334676	268.00000	43	7	269.00000	268.00000	0.37%	7.6	23523s
468331266	4334927	268.00000	47	11	269.00000	268.00000	0.37%	7.6	23526s
468392344	4335461	cutoff	46		269.00000	268.00000	0.37%	7.6	23529s
468451907	4335661	cutoff	57		269.00000	268.00000	0.37%	7.6	23532s
468512663	4335747	cutoff	71		269.00000	268.00000	0.37%	7.6	23535s
468573142	4336803	cutoff	52		269.00000	268.00000	0.37%	7.6	23538s
468634351	4337183	268.00000	68	11	269.00000	268.00000	0.37%	7.6	23541s
468691039	<b>4337566</b>	268.00000	70	5	269.00000	268.00000	0.37%	7.6	23544s
468748985	4336541	268.00000	62	16	269.00000	268.00000	0.37%	7.6	23547s
468805241	4335743	268.00000	65	18	269.00000	268.00000	0.37%	7.6	23550s
468866302	4335535	268.00000	66	9	269.00000	268.00000	0.37%	7.6	23553s
468926151	4334696	cutoff	65		269.00000	268.00000	0.37%	7.6	23556s
468985994	4334355	268.00000	63	14	269.00000	268.00000	0.37%	7.6	23559s
469044544	4333792	268.00000	52	9	269.00000	268.00000	0.37%	7.6	23562s
469097106	4333524	268.00000	72	8	269.00000	268.00000	0.37%	7.6	23565s
469156297	4332696	cutoff	83		269.00000	268.00000	0.37%	7.6	23568s

# Work Scheduling With Log Lines *(cont'd)*

*... then takes “forever” to prove optimality*

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
1330292347	2062	268.00000	45	14	269.00000	268.00000	0.37%	7.6	69930s
1330348816	1624	268.00000	79	22	269.00000	268.00000	0.37%	7.6	69933s
1330391972	920	268.00000	76	5	269.00000	268.00000	0.37%	7.6	69936s
1330448520	803	cutoff	69		269.00000	268.00000	0.37%	7.6	69939s
1330505973	333	268.00000	49	11	269.00000	268.00000	0.37%	7.6	69942s

Cutting planes:

Gomory: 1

Implied bound: 2

Thread count was 8 (of 8 available processors)

Times (seconds):

Input = 0.001

**Solve = 69944.7** (summed over threads)

Output = 0.012

Elapsed = 69944

Gurobi 3.0.0: **optimal solution; objective 269**

10114432447 simplex iterations

**1330555419 branch-and-cut nodes**

# Balanced Assignment: Complete Failure

*Starts well . . .*

	Nodes		Objective	IInf	Best Integer	Cuts/		ItCnt	Gap
	Node	Left				Best Node			
	0	0	0.0000	61		0.0000		99	
*	0+	0			232.0000	0.0000		99	100.00%
	0	0	0.0000	60	232.0000	Cuts: 55		174	100.00%
	0	0	0.0000	66	232.0000	Flowcuts: 17		250	100.00%
	0	0	0.0000	58	232.0000	Flowcuts: 9		300	100.00%
	0	0	0.0000	57	232.0000	Flowcuts: 13		326	100.00%
*	0+	0			230.0000	0.0000		326	100.00%
*	0+	0			216.0000	0.0000		326	100.00%
	0	2	0.0000	57	216.0000	0.0000		326	100.00%
*	440+	403			214.0000	0.0000		7938	100.00%
*	552+	339			212.0000	0.0000		10797	100.00%
	1000	556	69.9315	50	212.0000	0.0000		16491	100.00%
	2000	1332	42.8547	47	212.0000	0.0000		25669	100.00%
	3000	2276	81.6541	49	212.0000	5.0928		37332	97.60%
	4000	3214	77.9166	49	212.0000	5.1140		47933	97.59%
	5000	4160	71.0567	52	212.0000	6.4918		57582	96.94%
	6000	5089	97.3040	47	212.0000	7.8042		66662	96.32%
	7000	6021	158.4869	37	212.0000	9.3981		75348	95.57%
	8000	6942	157.5392	36	212.0000	11.2257		84237	94.70%
	.....								

# Balanced Assignment: Failure *(cont'd)*

*... bogs down without much further progress*

Node	Nodes Left	Objective	IInf	Best Integer	Cuts/ Best Node	ItCnt	Gap
.....							
6244000	5769420	91.8882	46	212.0000	55.4261	37227229	73.86%
6245000	5770348	123.4752	34	212.0000	55.4272	37233744	73.86%
6246000	5771270	63.5603	48	212.0000	55.4289	37239584	73.85%
6247000	5772192	106.5663	43	212.0000	55.4294	37245120	73.85%
6248000	5773112	64.0217	47	212.0000	55.4308	37251128	73.85%
6249000	5774034	181.2576	31	212.0000	55.4310	37257940	73.85%
6250000	5774954	119.4546	35	212.0000	55.4320	37263877	73.85%
Elapsed time = 9116.25 sec. (tree size = 1616.65 MB)							
Nodefile size = 1488.81 MB (685.88 MB after compression)							
6251000	5775885	182.0327	29	212.0000	55.4328	37270210	73.85%
6252000	5776807	140.1960	39	212.0000	55.4330	37275647	73.85%
6253000	5777720	91.9423	43	212.0000	55.4346	37281516	73.85%
6254000	5778648	127.8185	35	212.0000	55.4355	37286884	73.85%
8 flow-cover cuts							
2 Gomory cuts							
1 zero-half cut							
9 mixed-integer rounding cuts							
CPLEX 11.2.0: <b>ran out of memory.</b>							

# Stopping Early

## *1st example*

- ❖ optimum is either 269 or 268
- ❖ solution with 269 is known
- ❖ isn't that good enough?

## *2nd example*

- ❖ gap is still huge, but . . .
- ❖ solution with 212 seems likely to be optimal

*. . . we'll return to these*

## *The trouble with this reasoning*

- ❖ not robust over a range of similar problems
- ❖ could be wrong when gap is large

# 3: Multiprocess

## *Predictable effects*

- ❖ Process branch-and-bound nodes faster

## *Unpredictable effects*

- ❖ Build a different tree

## *Work scheduling examples . . .*

**least\_assign = 21** (*Gurobi 3.0 beta*)

*1 thread*

❖ 578661 nodes 213.322 seconds 0.369 sec / 1000 nodes

*8 threads*

❖ 132441 nodes 10.805 seconds 0.082 sec / 1000 nodes

*Speedup*

❖ 4.4 on nodes

❖ 4.5 on time/node

**least\_assign = 20** (*Gurobi 3.0 beta*)

*1 thread*

❖ 276903 nodes    93.374 seconds    0.337 sec / 1000 nodes

*8 threads*

❖ 186677 nodes    12.992 seconds    0.070 sec / 1000 nodes

*Speedup*

❖ 1.5 on nodes

❖ 4.8 on time/node

# least\_assign = 20 (CPLEX 12.1.0)

## *1 thread*

❖ 99342 nodes 54.766 seconds 0.551 sec / 1000 nodes

## *8 threads “deterministic”*

❖ 163140 nodes 16.847 seconds 0.103 sec / 1000 nodes

## *8 threads “opportunistic”*

❖ 62130 nodes 4.918 seconds 0.079 sec / 1000 nodes

❖ 165574 nodes 13.706 seconds 0.083 sec / 1000 nodes

❖ 153602 nodes 12.870 seconds 0.084 sec / 1000 nodes

❖ 310312 nodes 19.184 seconds 0.062 sec / 1000 nodes

❖ 599794 nodes 51.592 seconds 0.086 sec / 1000 nodes

❖ 154022 nodes 10.760 seconds 0.070 sec / 1000 nodes

❖ 75747 nodes 5.597 seconds 0.074 sec / 1000 nodes

❖ 383213 nodes 42.240 seconds 0.110 sec / 1000 nodes

# 4: Tune

*Default settings*

*Tuning run*

*Improved settings*

*Is tuning worthwhile?*

# Default Settings

## *CPLEX 11.1 run*

	Nodes					Cuts/		
	Node	Left	Objective	IInf	Best Integer	Best Node	ItCnt	Gap
	0	0	265.6000	22		265.6000	38	
	0	0	265.6000	22		Cuts: 13	72	
	0	0	265.6000	27		MIRcuts: 1	91	
*	0+	0			10348.0000	265.6000	91	97.43%
	0	2	265.6000	25	10348.0000	265.6000	91	97.43%
*	138	128	integral	0	289.0000	265.6000	3272	8.10%
*	140+	122			285.0000	265.6000	3292	6.81%
*	236	175	integral	0	283.0000	265.6000	4737	6.15%
*	318	168	integral	0	280.0000	265.6000	6175	5.14%
*	418+	140			272.0000	265.6000	8205	2.35%
*	418+	99			269.0000	265.6000	8205	1.26%
*	529+	120			267.0000	265.6000	9524	0.52%
*	85629+	1			266.0000	265.6000	1408458	0.15%

*159.2 seconds*

# Tuning Run

## *CPLEX tuning option*

```
ampl: option cplex_options 'tunefile t.out tunedisplay 2 tunetime 240';
ampl: solve;

CPLEX 11.1.0: tunefile t.out
tunedisplay 2
tunetime 240

Tuning on problem 'c252v269i126o126'

Test 'defaults':
  Integer optimal solution.
  Time = 155.31 sec. Objective = 266 Best bound = 266
Tuning progress: 13%

Test 'short1':
  Time limit exceeded.
  Time = 15.55 sec. Objective = 267 Best bound = 265.6
Tuning progress: 14%
```

# Tuning Run (*cont'd*)

## *CPLEX tuning option (cont'd)*

```
Test 'short_test2':
CPX_PARAM_CUTPASS 1
  Time limit exceeded.
  Time = 15.53 sec. Objective = 267 Best bound = 265.6
Tuning progress: 16%

Test 'short_test3':
CPX_PARAM_FRACCUTS 2
  Time limit exceeded.
  Time = 15.55 sec. Objective = 267 Best bound = 265.6
Tuning progress: 17%

Test 'short_test4':
CPX_PARAM_FRACPASS 10
CPX_PARAM_FRACCAND 10000
  Time limit exceeded.
  Time = 15.55 sec. Objective = 267 Best bound = 265.6
Tuning progress: 18%

.....
```

# Tuning Run (*cont'd*)

## *CPLEX tuning option (cont'd)*

```
Test 'short_test8':
CPX_PARAM_HEURFREQ  100
CPX_PARAM_CUTSFACTOR  100
CPX_PARAM_BTTOL  -1070056790
  Integer optimal solution.
  Time =    9.11 sec.  Objective = 266    Best bound = 266
Tuning progress: 49%
.....

Test 'short_test10':
CPX_PARAM_RINSHEUR  20
CPX_PARAM_HEURFREQ  3
CPX_PARAM_CUTSFACTOR  20
  Integer optimal solution.
  Time =    8.84 sec.  Objective = 266    Best bound = 266
Tuning progress: 53%
.....
```

# Tuning Run (*cont'd*)

## *CPLEX tuning option (cont'd)*

```
Test 'short_test8':
CPX_PARAM_HEURFREQ  100
CPX_PARAM_CUTSFACOR  100
CPX_PARAM_BTTOL    -1070056790
    Integer optimal solution.
    Time =      9.11 sec.  Objective = 266    Best bound = 266
Tuning progress: 49%
.....

Test 'short_test24':
CPX_PARAM_CUTPASS   1
CPX_PARAM_HEURFREQ -1
CPX_PARAM_PROBE    -1
CPX_PARAM_VARSEL   4
    Integer optimal solution.
    Time =      6.16 sec.  Objective = 266    Best bound = 266
Tuning progress: 95%
.....
```

# Tuning Run (*cont'd*)

## *CPLEX tuning option (conclusion)*

```
Tuning finished.  
4 settings written to tunefile file "t.out"  
ampl: option cplex_options 'paramfile t.out mipdisplay 2 mipinterval 1000';  
ampl: solve;  
CPLEX 11.1.0: cutpass = 1  
heurfreq = -1  
probe = -1  
varsel = 4
```

# Improved Settings

## *CPLEX 11.1 run*

Nodes		Cuts/						
Node	Left	Objective	IInf	Best Integer	Best Node	ItCnt	Gap	
	0	265.6000	22		265.6000	38		
	0	265.6000	22		Cuts: 13	72		
	0	265.6000	22		265.6000	72		
*	101	100	integral	0	322.0000	265.6000	1074	17.52%
*	162	138	integral	0	295.0000	265.6000	1621	9.97%
*	164	137	integral	0	291.0000	265.6000	1641	8.73%
*	229	187	integral	0	284.0000	265.6000	2540	6.48%
*	286	239	integral	0	283.0000	265.6000	3129	6.15%
*	352	292	integral	0	281.0000	265.6000	4397	5.48%
*	715	393	integral	0	272.0000	265.6000	10512	2.35%
*	1584	645	integral	0	269.0000	265.6000	26976	1.26%
*	3429	1545	integral	0	268.0000	265.6000	60651	0.90%
*	9877	0	integral	0	266.0000	265.6000	165097	0.15%

*6.45 seconds*

# Improved Settings (*cont'd*)

## *Results using mixed-integer formulation*

- ❖ With defaults: 159.2 sec, 85629 nodes, 1408458 iters
- ❖ After tuning: 6.45 sec, 9878 nodes, 165097 iters

## *Settings for tuned run*

- ❖ `cutpass = 1`
- ❖ `heurfreq = -1`
- ❖ `probe = -1`
- ❖ `varsel = 4`

*... same result with default of probe = 0*

# Is Tuning Worthwhile?

*Yes, sometimes*

- ❖ Default settings are not always so good

*But . . .*

- ❖ Chance may play a role
- ❖ No settings may be consistently better

*And you could always . . .*

# Try Another Solver

## *Gurobi 1.1 run (with default settings)*

Found heuristic solution: objective 408.0000

Root relaxation: objective 2.656000e+02, 42 iterations

Nodes		Current Node				Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time	
	0	0	265.6000	0	23	408.0000	265.6000	34.9%	-	0s
H	0	0				280.0000	265.6000	5.14%	-	0s
	0	0	266.0000	0	21	280.0000	266.0000	5.00%	-	0s
	0	0	266.0000	0	21	280.0000	266.0000	5.00%	-	0s
	0	0	266.0000	0	6	280.0000	266.0000	5.00%	-	0s
	0	2	266.0000	0	6	280.0000	266.0000	5.00%	-	0s
H	56	52				272.0000	266.0000	2.21%	14.7	0s
H	443	159				271.0000	266.0000	1.85%	6.6	0s
	833	319	266.0000	31	7	271.0000	266.0000	1.85%	8.5	1s
H	912	69				267.0000	266.0000	0.37%	8.8	1s
	2311	743	cutoff	99		267.0000	266.0000	0.37%	9.4	2s
H	4022	3				266.0000	266.0000	0.00%	8.5	2s

*2.83 seconds (2 threads)*

# Try Another Tuner

## *Paper at this year's CPAIOR conference*

- ❖ Frank Hutter, Holger H. Hoos, Kevin Leyton-Brown,  
**Automated Configuration of  
Mixed Integer Programming Solvers**
- ❖ We study the application of an automated algorithm configuration procedure to different MIP solvers, instance types and optimization objectives. We show that this fully-automated process yields substantial improvements to the performance of three MIP solvers: CPLEX, GUROBI, and LPSOLVE. Although our method can be used “out of the box” without any domain knowledge specific to MIP, we show that it outperforms the CPLEX special-purpose automated tuning tool.
- ❖ [www.cs.ubc.ca/labs/beta/Projects/MIP-Config/](http://www.cs.ubc.ca/labs/beta/Projects/MIP-Config/)

# 5: Reformulate

## *Balanced assignment revisited*

- ❖ Previous failed run
- ❖ Tighter formulations
- ❖ Successful run

## *Reconceived formulations*

# Failed Run

*Active start . . .*

	Nodes				Cuts/			
	Node	Left	Objective	IInf	Best Integer	Best Node	ItCnt	Gap
	0	0	0.0000	61		0.0000	99	
*	0+	0			232.0000	0.0000	99	100.00%
	0	0	0.0000	60	232.0000	Cuts: 55	174	100.00%
	0	0	0.0000	66	232.0000	Flowcuts: 17	250	100.00%
	0	0	0.0000	58	232.0000	Flowcuts: 9	300	100.00%
	0	0	0.0000	57	232.0000	Flowcuts: 13	326	100.00%
*	0+	0			230.0000	0.0000	326	100.00%
*	0+	0			216.0000	0.0000	326	100.00%
	0	2	0.0000	57	216.0000	0.0000	326	100.00%
*	440+	403			214.0000	0.0000	7938	100.00%
*	552+	339			212.0000	0.0000	10797	100.00%
	1000	556	69.9315	50	212.0000	0.0000	16491	100.00%
	2000	1332	42.8547	47	212.0000	0.0000	25669	100.00%
	3000	2276	81.6541	49	212.0000	5.0928	37332	97.60%
	4000	3214	77.9166	49	212.0000	5.1140	47933	97.59%
	5000	4160	71.0567	52	212.0000	6.4918	57582	96.94%
	6000	5089	97.3040	47	212.0000	7.8042	66662	96.32%
	7000	6021	158.4869	37	212.0000	9.3981	75348	95.57%
	8000	6942	157.5392	36	212.0000	11.2257	84237	94.70%
	.....							

# Failed Run (*cont'd*)

*... bogs down completely*

Node	Nodes Left	Objective	IInf	Best Integer	Cuts/ Best Node	ItCnt	Gap
.....							
6244000	5769420	91.8882	46	212.0000	55.4261	37227229	73.86%
6245000	5770348	123.4752	34	212.0000	55.4272	37233744	73.86%
6246000	5771270	63.5603	48	212.0000	55.4289	37239584	73.85%
6247000	5772192	106.5663	43	212.0000	55.4294	37245120	73.85%
6248000	5773112	64.0217	47	212.0000	55.4308	37251128	73.85%
6249000	5774034	181.2576	31	212.0000	55.4310	37257940	73.85%
6250000	5774954	119.4546	35	212.0000	55.4320	37263877	73.85%
Elapsed time = 9116.25 sec. (tree size = 1616.65 MB)							
Nodefile size = 1488.81 MB (685.88 MB after compression)							
6251000	5775885	182.0327	29	212.0000	55.4328	37270210	73.85%
6252000	5776807	140.1960	39	212.0000	55.4330	37275647	73.85%
6253000	5777720	91.9423	43	212.0000	55.4346	37281516	73.85%
6254000	5778648	127.8185	35	212.0000	55.4355	37286884	73.85%
8 flow-cover cuts							
2 Gomory cuts							
1 zero-half cut							
9 mixed-integer rounding cuts							
CPLEX 11.2.0: ran out of memory.							

# Tighter Formulation 1

## *Definition of overlap for person i*

```
minimize TotalOverlap:
    sum {i in PEOPLE} Overlap[i];

subj to OverlapDefn {i in PEOPLE, j in 1..numberGrps}:
    Overlap[i] >=
        sum {i2 in PEOPLE diff {i}: title[i2] = title[i]} Assign[i2,j] +
        sum {i2 in PEOPLE diff {i}: loc[i2] = loc[i]} Assign[i2,j] +
        sum {i2 in PEOPLE diff {i}: dept[i2] = dept[i]} Assign[i2,j] +
        sum {i2 in PEOPLE diff {i}: sex[i2] = sex[i]} Assign[i2,j]
        - maxOverlap[i] * (1 - Assign[i,j]);
```

- $\text{maxOverlap}[i]$  must be  $\geq$  greatest overlap possible
- Smaller values give stronger b&b lower bounds
  - \* theoretically correct:  $4 * (\text{maxInGrp}-1) \rightarrow 0.0$
  - \* empirically justified:  $1 * (\text{maxInGrp}-1) \rightarrow 156.8 (26.0\%)$

# Tighter Formulation 2

## *Symmetry constraint (a)*

```
subject to BreakSymm1
    {i in FIRST_PEOPLE, j in ord(i,FIRST_PEOPLE)+1..numberGrps}:
        Assign[i,j] = 0;
```

- choose the first ( $\text{numberGrps}-1$ ) people in some way
- assign the  $i$ th person to one of the first  $i$  groups

# Tighter Formulation 2 (*cont'd*)

## *Symmetry constraint (b)*

```
set TYPES = setof {i in PEOPLE} (title[i],loc[i],dept[i],sex[i]);
set TYPEpeople {(t1,t2,t3,t4) in TYPES} =
  {i in PEOPLE: title[i]=t1 and loc[i]=t2 and
    dept[i]=t3 and sex[i]=t4} ordered by PEOPLE;

subject to BreakSymm2 {(t1,t2,t3,t4) in TYPES,
  pnum in 1..card(TYPEpeople[t1,t2,t3,t4])-1, j in 1..numberGrps, k in 1..j-1}:
  Assign[member(pnum+1,TYPEpeople[t1,t2,t3,t4]),k]
    <= 1 - Assign[member(pnum,TYPEpeople[t1,t2,t3,t4]),j];
```

- identify “types” of people who are identical in all four characteristics
- order the people of each type, and order the groups
- with each type, assign higher-numbered people to higher-numbered groups

# Tighter Formulation 2 (*cont'd*)

## *Symmetry strategies*

- BreakSymm1 increases the b&b lower bound a bit
- BreakSymm2 does not increase the lower bound
- CPLEX's symmetry directive is more effective
  - \* set symmetry=5 for greatest symmetry-breaking effort

# Tighter Formulation 3

## *Group size limits*

```
subj to GroupSize {j in 1..numberGrps}:  
    minInGrp <= sum {i in PEOPLE} Assign[i,j] <= maxInGrp;
```

- miningrp must be smaller than group size average
- maxingrp must be larger than group size average
- Tighter limits give stronger b&b lower bounds
  - \*  $\text{floor}(\text{card}(\text{PEOPLE})/\text{numberGrps}) - 1$   
     $\text{ceil}(\text{card}(\text{PEOPLE})/\text{numberGrps}) + 1 \rightarrow 156.8 \text{ (26.0\%)}$
  - \*  $\text{floor}(\text{card}(\text{PEOPLE})/\text{numberGrps})$   
     $\text{ceil}(\text{card}(\text{PEOPLE})/\text{numberGrps}) \rightarrow 177.6 \text{ (16.2\%)}$

# Tighter Formulation 3 (*cont'd*)

## *Group sizes*

```
param minInGrp := floor (card(PEOPLE)/numberGrps);
param nMinInGrp := numberGrps - card{PEOPLE} mod numberGrps;

subj to GroupSizeMin {j in 1..nMinInGrp}:
    sum {i in PEOPLE} Assign[i,j] = minInGrp;

subj to GroupSizeMax {j in nMinInGrp+1..numberGrps}:
    sum {i in PEOPLE} Assign[i,j] = minInGrp + 1;
```

### ➤ Compute exact sizes of all groups

\* min, max sizes → 177.6  
(16.2%)

\* exact sizes → 183.36  
(13.5%)

### Example 3

# Balanced Assignment

## *Incorporating enhancements . . .*

```
ampl: model gs1f.mod;
ampl: data gs1b.dat;
ampl: option solver cplex;
ampl: option cplex_options 'symmetry 5 mipdisplay 2 mipinterval 1000';
ampl: solve;
```

MIP Presolve eliminated 54 rows and 0 columns.

MIP Presolve modified 2636 coefficients.

Reduced MIP has 197 rows, 156 columns, and 2585 nonzeros.

Reduced MIP has 130 binaries, 0 generals, 0 SOSs, and 0 indicators.

Clique table members: 62.

MIP emphasis: balance optimality and feasibility.

MIP search method: dynamic search.

Parallel mode: none, using 1 thread.

Root relaxation solution time = 0.03 sec.

	Nodes					Cuts/		
	Node	Left	Objective	IInf	Best Integer	Best Node	ItCnt	Gap
*	0+	0			252.0000		0	---
	0	0	183.3626	134	252.0000	183.3626	262	27.24%
.....								

# Successful Run

*Much more promising start . . .*

	Nodes					Cuts/		
	Node	Left	Objective	IInf	Best Integer	Best Node	ItCnt	Gap
	0	0	189.1865	100	252.0000	Cuts: 49	445	24.93%
	0	0	189.7246	96	252.0000	Cuts: 12	558	24.71%
*	0+	0			240.0000	189.7246	558	20.95%
	0	0	189.7964	96	240.0000	ZeroHalf: 5	664	20.92%
	0	0	189.8864	97	240.0000	ZeroHalf: 8	782	20.88%
	0	0	189.9590	96	240.0000	ZeroHalf: 6	1002	20.85%
	0	0	189.9768	100	240.0000	ZeroHalf: 7	1166	20.84%
	0	0	189.9769	99	240.0000	ZeroHalf: 4	1184	20.84%
*	0+	0			220.0000	189.9769	1203	13.65%
*	0+	0			216.0000	189.9769	1203	12.05%
	0	2	192.8299	78	216.0000	192.8299	1203	10.73%
*	100+	80			212.0000	193.0563	6092	8.94%
	1000	479	200.3732	83	212.0000	195.6130	36233	7.73%
	2000	1242	205.1626	64	212.0000	195.9832	65307	7.56%
	3000	2103	205.8520	59	212.0000	196.4174	93546	7.35%
	4000	2946	205.5224	57	212.0000	196.8495	120479	7.15%
	5000	3790	201.5651	53	212.0000	197.1664	145209	7.00%
	6000	4624	210.5546	34	212.0000	197.4648	169658	6.86%
	7000	5468	201.2841	60	212.0000	197.6005	195286	6.79%
	.....							

# Successful Run (*cont'd*)

*... leads to successful conclusion*

Node	Nodes Left	Objective	IInf	Best Integer	Cuts/ Best Node	ItCnt	Gap
30287000	8802	cutoff		212.0000	211.0000	416705257	0.47%
30288000	7927	cutoff		212.0000	211.0000	416709767	0.47%
30289000	7021	infeasible		212.0000	211.0000	416714199	0.47%
30290000	6101	infeasible		212.0000	211.0000	416718973	0.47%
Elapsed time = 46415.00 sec. (tree size = 12.94 MB)							
30291000	5249	cutoff		212.0000	211.0000	416724639	0.47%
30292000	4407	infeasible		212.0000	211.0000	416730198	0.47%
30293000	3519	infeasible		212.0000	211.0000	416735118	0.47%
30294000	2636	cutoff		212.0000	211.0000	416740781	0.47%
30295000	1758	infeasible		212.0000	211.0000	416746255	0.47%
30296000	863	infeasible		212.0000	211.0000	416748900	0.47%
3 cover cuts							
8 implied bound cuts							
23 mixed-integer rounding cuts							
35 zero-half cuts							
12 Gomory fractional cuts							
CPLEX 11.2.0: optimal integer solution; objective 212							
416751729 MIP simplex iterations							
30296965 branch-and-bound nodes							

# Reconceived Formulation

## *Different variables*

- Min of each type in any group
- Max of each type in any group

## *Different objective*

- Sum of (max – min) over all types

## 6: “Cheat” on the method

*Work scheduling revisited . . .*

*Cover demands for workers*

- ❖ Each “shift” requires a certain number of employees
- ❖ Each employee works a certain “schedule” of shifts
- ❖ *Each schedule that is worked by anyone must be worked by a fixed minimum number*

*Minimize total workers needed*

- ❖ Which schedules are used? **Use[j]** vars
- ❖ How many work each of schedule? **Work[j]** vars

# Work Scheduling Revisited

## *Direct approach*

- ❖ Apply branch-and-bound to whole problem
- ❖ Branch “up” first

## *Indirect approach*

- ❖ Step 1: Relax integrality of **Work** variables  
Solve for zero-one **Use** variables
- ❖ Step 2: Fix **Use** variables  
Solve for integer **Work** variables

*. . . not necessarily optimal, but . . .*

# Work Scheduling Revisited

## *Typical run of indirect approach*

```
ampl: model sched1.mod; data sched.dat;
ampl: let least_assign := 16;
ampl: option solver cplex;
ampl: option cplex_options 'branch 1';
ampl: let {j in SCHEDS} Work[j].relax := 1;
ampl: solve;

CPLEX 11.2.0: optimal integer solution; objective 265.6
870496 MIP simplex iterations
55911 branch-and-bound nodes

ampl: fix {j in SCHEDS} Use[j];
ampl: let {j in SCHEDS} Work[j].relax := 0;
ampl: solve;

CPLEX 11.2.0: optimal integer solution; objective 266
24 MIP simplex iterations
4 branch-and-bound nodes
```

# Work Scheduling: Hard Case

## *CPLEX 12.1*

- ❖ Direct: 465,596,558 nodes, 112013 seconds
- ❖ Indirect: 6,886,122 nodes, 617 seconds

## *Gurobi 3.0 beta*

- ❖ Direct: 1,330,555,419 nodes, 69945 seconds
- ❖ Indirect: 6,354,683 nodes, 299 seconds

## *Observations*

- ❖ step 1 gives fractional solution
- ❖ step 2 trivially easy and rounds up step 1 objective

*... hence optimal*

# 7: “Cheat” on the data

## *Cut large “raw” rolls into smaller ones*

- ❖ All raw rolls the same width
- ❖ Various smaller widths ordered
- ❖ Varying numbers of widths ordered

## *Minimize total raw rolls cut*

- ❖ By generating patterns during optimization
- ❖ By enumerating patterns in advance

# Roll Cutting

## *Cutting model*

```
set WIDTHS;                                # set of widths to be cut
param orders {WIDTHS} > 0;                  # number of each width to be cut

param nPAT integer >= 0;                    # number of patterns
param nbr {WIDTHS,1..nPAT} integer >= 0;  # rolls of width i in pattern j

var Cut {1..nPAT} integer >= 0;           # rolls cut using each pattern

minimize Number:

    sum {j in 1..nPAT} Cut[j];             # total raw rolls cut

subject to Fill {i in WIDTHS}:

    sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];

                                            # for each width,
                                            # rolls cut meet orders
```

# Roll Cutting (*cont'd*)

## *Pattern generation model*

```
param roll_width > 0;
param price {WIDTHS} default 0.0;
var Use {WIDTHS} integer >= 0;

minimize Reduced_Cost:
    1 - sum {i in WIDTHS} price[i] * Use[i];

subj to Width_Limit:
    sum {i in WIDTHS} i * Use[i] <= roll_width;
```

# Roll Cutting (*cont'd*)

## *Pattern generation script*

```
repeat {  
  solve Cutting_Opt;  
  let {i in WIDTHS} price[i] := Fill[i].dual;  
  solve Pattern_Gen;  
  if Reduced_Cost < -0.00001 then {  
    let nPAT := nPAT + 1;  
    let {i in WIDTHS} nbr[i,nPAT] := Use[i];  
  }  
  else break;  
};
```

# Roll Cutting (*cont'd*)

## *Pattern enumeration script*

```
repeat {
  if curr_sum + curr_width <= roll_width then {
    let pattern[curr_width] := floor((roll_width-curr_sum)/curr_width);
    let curr_sum := curr_sum + pattern[curr_width] * curr_width;
  }

  if curr_width != last(WIDTHS) then
    let curr_width := next(curr_width,WIDTHS);
else {
  let nPAT := nPAT + 1;
  let {w in WIDTHS} nbr[w,nPAT] := pattern[w];
  let curr_sum := curr_sum - pattern[last(WIDTHS)] * last(WIDTHS);
  let pattern[last(WIDTHS)] := 0;
  let curr_width := min {w in WIDTHS: pattern[w] > 0} w;
  if curr_width < Infinity then {
    let curr_sum := curr_sum - curr_width;
    let pattern[curr_width] := pattern[curr_width] - 1;
    let curr_width := next(curr_width,WIDTHS);
  }

  else break;
}
}
```

# Roll Cutting (*cont'd*)

## *Sample data*

```
param roll_width := 172 ;
param: WIDTHS: orders :=
    25.000    5
    24.750    73
    18.000    14
    17.500    4
    15.500    23
    15.375    5
    13.875    29
    12.500    87
    12.250    9
    12.000    31
    10.250    6
    10.125    14
    10.000    43
    8.750     15
    8.500     21
    7.750     5 ;
```

*. . . Robert W. Haessler, “Selection  
and Design of Heuristic Procedures  
for Solving Roll Trim Problems”  
Management Science 34 (1988)  
1460–1471, Table 2*

# Roll Cutting Results

*Patterns generated during optimization  
(Gilmore-Gomory procedure)*

- 32.80 rolls in continuous relaxation
- 40 rolls rounded up to integer
- 34 rolls solving IP using generated patterns

*All patterns enumerated in advance*

- 27,338,021 non-dominated patterns — too big

*Every 100<sup>th</sup> pattern saved*

- 273,380 patterns
- 33 rolls solving IP using enumerated patterns
- 50 seconds: b&b heuristic solves at root (no cuts)

*. . . takes much longer to generate than solve*