

New Directions in the Development of AMPL



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New Directions in Development of the AMPL Modeling Language and System

We summarize current and planned projects to extend and enhance AMPL, with the aim of facilitating both formulation and implementation of optimization models. Extensions to AMPL's language are designed to allow for more natural description of discrete models and stochastic data. New solver interfaces will make nontraditional solvers more accessible for practical modeling. A new callable interface to the AMPL system will facilitate business deployment.

AMPL

Algebraic modeling language: symbolic data

```
set SHIFTS;                # shifts
param Nsched;             # number of schedules;
set SCHEDS = 1..Nsched;  # set of schedules

set SHIFT_LIST {SCHEDS} within SHIFTS;

param rate {SCHEDS} >= 0;    # pay rates
param required {SHIFTS} >= 0; # staffing requirements
param least_assign >= 0;    # min workers on any schedule used
```

AMPL

Algebraic modeling language: symbolic model

```
var Work {SCHEDS} >= 0 integer;
var Use  {SCHEDS} >= 0 binary;

minimize Total_Cost:
    sum {j in SCHEDS} rate[j] * Work[j];

subject to Shift_Needs {i in SHIFTS}:
    sum {j in SCHEDS: i in SHIFT_LIST[j]} Work[j] >= required[i];

subject to Least_Use1 {j in SCHEDS}:
    least_assign * Use[j] <= Work[j];

subject to Least_Use2 {j in SCHEDS}:
    Work[j] <= (max {i in SHIFT_LIST[j]} required[i]) * Use[j];
```

AMPL

Explicit data independent of symbolic model

```
set SHIFTS := Mon1 Tue1 Wed1 Thu1 Fri1 Sat1
             Mon2 Tue2 Wed2 Thu2 Fri2 Sat2
             Mon3 Tue3 Wed3 Thu3 Fri3 ;

param Nsched := 126 ;

set SHIFT_LIST[1] := Mon1 Tue1 Wed1 Thu1 Fri1 ;
set SHIFT_LIST[2] := Mon1 Tue1 Wed1 Thu1 Fri2 ;
set SHIFT_LIST[3] := Mon1 Tue1 Wed1 Thu1 Fri3 ;
set SHIFT_LIST[4] := Mon1 Tue1 Wed1 Thu1 Sat1 ;
set SHIFT_LIST[5] := Mon1 Tue1 Wed1 Thu1 Sat2 ; .....

param required := Mon1 100 Mon2 78 Mon3 52
                  Tue1 100 Tue2 78 Tue3 52
                  Wed1 100 Wed2 78 Wed3 52
                  Thu1 100 Thu2 78 Thu3 52
                  Fri1 100 Fri2 78 Fri3 52
                  Sat1 100 Sat2 78 ;
```

AMPL

Solver independent of model & data

```
ampl: model sched1.mod;
ampl: data sched.dat;

ampl: let least_assign := 7;

ampl: option solver cplex;
ampl: solve;

CPLEX 12.1.0: optimal integer solution; objective 266
473 MIP simplex iterations
72 branch-and-bound nodes

ampl: option omit_zero_rows 1, display_1col 0;
ampl: display Work;

Work [*] :=
  2 12    16 14    29  7    53  7    91 17    112  9    122 29
  3  7    18  7    37 21    78 21    100 19    116 20    124  7
  6 10    20  7    41  8    82 21    109  7    118 16

;
```

AMPL

Language independent of solver

```
ampl: option solver gurobi;  
ampl: solve;  
  
Gurobi 3.0.0: optimal solution; objective 266  
396 simplex iterations  
3 branch-and-cut nodes  
  
ampl: display Work;  
  
Work [*] :=  
  1 29    37 35    84 18    91 17    101 11    112 18    118 17  
 21 36    71  7    89 18    95  7    109 10    116  7    124 36  
;  

```

Topics

The company

- ❖ People
- ❖ Business developments

The language

- ❖ Varied prospective enhancements
- ❖ More natural formulations

The solvers

- ❖ Conic programming
- ❖ Nontraditional alternatives

The system

- ❖ APIs & IDEs
- ❖ AMPL as a service (in the cloud)

The Company

Background

- ❖ AMPL at Bell Labs (1986)
- ❖ AMPL commercialization (1993)
- ❖ AMPL Optimization LLC (2002)

Developments

- ❖ People
- ❖ Business

People

Bob Fourer

❖ Founder & . . .

Dave Gay

❖ Founder & . . .

Bill Wells

❖ Director of business development

Business Developments

AMPL intellectual property

- ❖ Full rights acquired from Alcatel-Lucent USA
 - * corporate parent of Bell Laboratories
- ❖ More flexible licensing terms available

CPLEX with AMPL

- ❖ Sales transferred from IBM to AMPL Optimization
- ❖ Full lineup of licensing arrangements available

AMPL distributors

- ❖ New for Japan: *October Sky Co., Ltd.* →
- ❖ Others continue active
 - * Gurobi, Ziena
 - * MOSEK, TOMLAB
 - * OptiRisk



AMPL

AMPL

最強の最適化モデリング言語
究極のスケールビリティ

AMPLは、スケールビリティが要求される大規模な最適化問題を効率的にモデル化するための代替モデリング言語です。簡単な問題をモデル化できるだけのモデリング言語が豊富にある中、AMPLは、現実社会の業務問題を解決するためのプロフェッショナル用のモデリング ツールと見えます。特に、集合を効率的に操作するための高度な豊富さと柔軟さは、他のモデリング言語の追従を許しません。また、ネットワークおよび区分的線形関数を入力するための特別な表記法も組み込んでおり、ユーザー数の多さ、そして、長年の開発の積み重ねによって、極めて使い易いインターフェイスをユーザに提供します。AMPLは、まさに最強のモデリング言語です。



AMPL MEANS BUSINESS

The Language

Background

- ❖ Power & convenience
 - * Linear and nonlinear modeling
 - * Extensive indexing and set expressions
- ❖ Prototyping & deployment
 - * Integrated scripting language
- ❖ Business & research
 - * Major installations worldwide
 - * Hundreds of citations in scientific & engineering literature

Plans . . .

The Language

Plans

- ❖ Further set operations
 - * arg min/arg max
 - * sort set by parameter values
 - * arbitrary selection from an unordered set
- ❖ Random parameters/variables
 - * send as input to stochastic solvers
- ❖ Enhanced scripting
 - * faster loops
 - * functions defined by scripts
- ❖ *More natural formulations . . .*

Common Areas of Confusion

Examples from my e-mail . . .

- ❖ I have been trying to write a stepwise function in AMPL but I have not been able to do so:

```
fc[wh] = 100 if x[wh] <=5
        300 if 6 <= x[wh] <=10
        400 if 11 <= x[wh]
```

where fc and x are variables.

- ❖ *I have a set of nonlinear equations to be solved, and variables are binary. Even I have an xor operator in the equations. How can I implement it and which solver is suitable for it?*
- ❖ I'm a recent IE grad with just one grad level IE course under my belt. . . .

```
minimize Moves: sum{emp in GROUPA}
              (if Sqrt((XEmpA[emp] - XGrpA)^2 +
                    (YEmpA[emp] - YGrpA)^2) > Ra then 1 else 0)
```

Is there some documentation on when you can and cannot use the if-then statements in AMPL (looked through the related forum posts but still a bit confused on this)?

Common Areas of Confusion

Examples from my e-mail (cont'd)

- ❖ I have a problem need to add a such kind of constraint:

Max[$\sum(P_i * H_i)$]; i is from 1 to 24;

in which P_i are constant and H_i need to be optimized.

Bound is $-180 \leq H_i \leq 270$. One of the constraints is

$\sum(C_i) = 0$; here $C_i = H_i$ if $H_i > 0$ and $C_i = H_i/1.38$ if $H_i < 0$

Is it possible to solve this kind of problem with `lp_solve`?
and how to setup the constraint?

- ❖ *... is there a way to write a simple "or" statement in AMPL like in Java or C++?*
- ❖ I need to solve the following optimization problem:

Minimize $-|x_1| - |x_2|$

subject to

$x_1 - x_2 = 3$

Do you know how to transform it to standard linear program?

More Natural Formulations

Currently Implemented

Extension to mixed-integer solver

- ❖ CPLEX indicator constraints
 - * `Use[j] = 1 ==> Work[j] >= least_assign;`

Translation to mixed-integer programs

- ❖ General variable domains
 - * `var Work {j in SCHEDULES} integer,
in {0} union interval[lo_assign, hi_assign];`
- ❖ Separable piecewise-linear terms
 - * `<<avail_min[t]; 0,time_penalty[t]>> Use[t]`

Translation to general nonlinear programs

- ❖ Complementarity conditions
 - * `0 <= ct[cr,u] complements
ctcost[cr,u] + cv[cr] >= p["C",u];`

More Natural Formulations

Prospective Extensions

Existing operators allowed on variables

- ❖ Nonsmooth terms
- ❖ Conditional expressions

New forms

- ❖ Operators on constraints
- ❖ New aggregate operators
- ❖ Generalized indexing: variables in subscripts
- ❖ New types of variables: object-valued, set-valued

Solution strategies

- ❖ Transform to standard MIPs
- ❖ *Send to alternative solvers (will return to this)*

Extensions

Piecewise-Linear Terms

Transportation (multiple rates)

```
minimize Total_Cost:  sum {i in ORIG, j in DEST}
  << limit1[i,j], limit2[i,j];
  rate1[i,j], rate2[i,j], rate3[i,j] >> Trans[i,j];
```

```
minimize Total_Cost:  sum {i in ORIG, j in DEST}
  << {p in 1..npiece[i,j]-1} limit[i,j,p];
  {p in 1..npiece[i,j]} rate[i,j,p] >> Trans[i,j];
```

Production (overtime)

```
maximize Total_Profit:  sum {p in PROD, t in 1..T}
  (rev[p,t]*Sell[p,t] - pcost[p]*Make[p,t] - icost[p]*Inv[p,t]) -
  sum {t in 1..T} << avail_min[t]; 0,time_penalty[t] >> Use[t];
```

General Variable Domains

Workforce Scheduling

```
param least_assign >= 0;  
  
var Work {j in SCHEDS} integer, in {0} union  
    interval [least_assign, (max {i in SHIFT_LIST[j]} required[i])];
```

Extensions

Logical Operators

Flow shop scheduling

```
subj to NoConflict {i1 in JOBS, i2 in JOBS: ord(i1) < ord(i2)}:  
    Start[i2] >= Start[i1] + setTime[i1,i2] or  
    Start[i1] >= Start[i2] + setTime[i2,i1];
```

Balanced assignment

```
subj to NoIso {(i1,i2) in TYPE, j in ROOM}:  
    not (Assign[i1,i2,j] = 1 and  
        sum {ii1 in ADJ[i1]: (ii1,i2) in TYPE} Assign[ii1,i2,j] = 0);
```

Location-transportation

```
subj to Capacity {i in WHSE}:  
    if Build[i] = 1  
        then sum {j in CUST} Ship[i,j] <= cap[i]  
        else forall {j in CUST} Ship[i,j] = 0;
```

Extensions

Implication Operator

Multicommodity flow with fixed costs

```
subject to DefineUsedA {i in ORIG, j in DEST}:  
    Use[i,j] = 0 ==> sum {p in PROD} Trans[i,j,p] = 0;
```

```
subject to DefineUsedB {i in ORIG, j in DEST, p in PROD}:  
    Use[i,j] = 0 ==> Trans[i,j,p] = 0;
```

Workforce planning

```
var NoShut {m in MONTHS} binary;  
var LayoffCost {m in MONTHS} >=0;  
  
subj to NoShutDefn1 {m in MONTHS}:  
    NoShut[m] = 1 ==> LayoffCost[m] = 0;  
  
subj to NoShutDefn2 {m in MONTHS}:  
    NoShut[m] = 0 ==> LayoffCost[m] =  
        snrLayoffWages * ShutdownDays[m] * maxNumberSnrEmpl;
```

Extensions

Counting Operators

Transportation

```
subj to MaxServe {i in ORIG}:  
  card {j in DEST: sum {p in PRD} Trans[i,j,p] > 0} <= mxsrv;
```

```
subj to MaxServe {i in ORIG}:  
  count {j in DEST} (sum {p in PRD} Trans[i,j,p] > 0) <= mxsrv;
```

```
subj to MaxServe {i in ORIG}:  
  atmost mxsrv {j in DEST} (sum {p in PRD} Trans[i,j,p] > 0);
```

Extensions

“Structure” Operators

Assignment

```
subj to OneJobPerMachine:  
    alldiff {j in JOBS} (MachineForJob[j]);
```

```
subj to CapacityOfMachine {k in MACHINES}:  
    numberof k {j in JOBS} (MachineForJob[j]) <= cap[k];
```

. . . argument in () may be a more general list

Extensions

Variables in Subscripts

Assignment

```
minimize TotalCost:  
    sum {j in JOBS} cost[j,MachineForJob[j]];
```

Sequencing

```
minimize CostPlusPenalty:  
    sum {k in 1..nSlots} setupCost[JobForSlot[k-1],JobForSlot[k]] +  
    sum {j in 1..nJobs} duePen[j] * (dueTime[j] - ComplTime[j]);  
  
subj to TimeNeeded {k in 0..nSlots-1}:  
    ComplTime[JobForSlot[k]] =  
        min( dueTime[JobForSlot[k]],  
            ComplTime[JobForSlot[k+1]]  
            - setupTime[JobForSlot[k],JobForSlot[k+1]]  
            - procTime[JobForSlot[k+1]] );
```

Extensions

Object-Valued Variables

Location

```
set CLIENTS;
set WHSES;

param srvCost {CLIENTS, WHSES} > 0;
param bdgCost > 0;

var Serve {CLIENTS} in WHSES;
var Open {WHSES} binary;

minimize TotalCost:
    sum {i in CLIENTS} srvCost[i, Serve[i]] +
    bdgcost * sum {j in WHSES} Open[j];

subject to OpenDefn {i in CLIENTS}:
    Open[Serve[i]] = 1;
```

Extensions

Set-Valued Variables

Crew scheduling

```
set SKILLset {SKILLS} within STAFF;
var CREWset {FLIGHTS} within STAFF;
. . . . .
subject to CrewSize {j in FLIGHTS}:
    card (CREWset[j]) = nbCrew[j];
subject to SkillReq {i in SKILLS, j in FLIGHTS}:
    card (SKILLset[i] inter CREWset[j]) >= nbSkills[i,j];
subject to NonConsecutive {j in FLIGHTS}:
    CREWset[j] inter CREWset[next(j)] = { };
```

The Solvers

Communication while solver is active

- ❖ Speed up multiple solves
- ❖ Support callbacks

Conic programming

- ❖ Barrier solvers available
- ❖ Stronger modeling support needed

Nontraditional alternatives

- ❖ Global optimization
- ❖ Constraint programming
- ❖ Varied hybrids

Conic Programming

Simple convex quadratic constraints

- ❖ Ball: $x_1^2 + \dots + x_n^2 \leq b$
- ❖ Cone: $x_1^2 + \dots + x_n^2 \leq y^2, y \geq 0$
- ❖ Cone: $x_1^2 + \dots + x_n^2 \leq yz, y \geq 0, z \geq 0$

... variables can be generalized to linear terms

Similarities

- ❖ Describe by lists of coefficients
- ❖ Solve by extensions of LP barrier methods; extend to MIP

Differences

- ❖ Quadratic part not positive semi-definite
- ❖ Nonnegativity is essential
- ❖ ***Many convex problems can be reduced to these ...***

Equivalent Problems: Minimize

Sums of . . .

- ❖ norms & squared norms
- ❖ norms / linear terms

Max of . . .

- ❖ norms
- ❖ logarithmic Chebychev terms
 - * $\max_i |\log(a_i x) - \log(b_i)|$

Product of . . .

- ❖ negative powers
 - * $\prod_i (a_i x + b_i)^{-\alpha_i}$ for rational $\alpha > 0$
- ❖ minus positive powers

. . . and certain sum-max combinations

Equivalent Problems: Subject to

Similar expressions involving

- ❖ norms & squared norms
- ❖ norms / linear terms
- ❖ negative & minus positive powers
- ❖ minus positive powers

. . . see Jared Erickson's talk in MC33.1 for details

Modeling SOCPs

Current situation

- ❖ each solver recognizes some elementary forms
- ❖ modeler must convert to these forms

Goal

- ❖ recognize many equivalent forms
- ❖ automatically convert to a canonical form
- ❖ further convert as necessary for each solver

Example: Sum of Norms

```
param p integer > 0;
param m {1..p} integer > 0;
param n integer > 0;

param F {i in 1..p, 1..m[i], 1..n};
param g {i in 1..p, 1..m[i]};
```

```
param p := 2 ;
param m := 1 5 2 4 ;
param n := 3 ;

param g (tr): 1 2 :=
    1 12 2
    2 7 11
    3 7 1
    4 8 0
    5 4 . ;

param F := ...
```

Example: Original Formulation

```
var x {1..n};  
minimize SumOfNorms:  
    sum {i in 1..p} sqrt(  
        sum {k in 1..m[i]} (sum {j in 1..n} F[i,k,j] * x[j] + g[i,k])^2 );
```

3 variables, all nonlinear
0 constraints
1 nonlinear objective; 3 nonzeros.

CPLEX 12.2.0.0: at12228.nl **contains a nonlinear objective.**

Example: Converted to Quadratic

```
var x {1..n};
var Max {1..p};

minimize SumOfNorms: sum {i in 1..p} Max[i];

subj to MaxDefinition {i in 1..p}:
    Max[i]^2 >=
        sum {k in 1..m[i]} (sum {j in 1..n} F[i,k,j] * x[j] + g[i,k])^2;
```

5 variables, all nonlinear
2 constraints, all nonlinear; 8 nonzeros
1 linear objective; 2 nonzeros.

CPLEX 12.2.0.0: **QP Hessian is not positive semi-definite.**

Example: Simpler Quadratic

```
var x {1..n};
var Max {1..p} >= 0;
var Fxplusg {i in 1..p, 1..m[i]};
minimize SumOfNorms: sum {i in 1..p} Max[i];
subj to MaxDefinition {i in 1..p}:
    Max[i]^2 >= sum {k in 1..m[i]} Fxplusg[i,k]^2;
subj to FxplusgDefinition {i in 1..p, k in 1..m[i]}:
    Fxplusg[i,k] = sum {j in 1..n} F[i,k,j] * x[j] + g[i,k];
```

```
14 variables:
    11 nonlinear variables
    3 linear variables
11 constraints; 41 nonzeros
    2 nonlinear constraints
    9 linear constraints
1 linear objective; 2 nonzeros.
```

CPLEX 12.2.0.0: primal optimal; objective 11.03323293; 11 barrier iters

Nontraditional Solvers

Global nonlinear

- ❖ BARON *
- ❖ LINDO Global *
- ❖ LGO

Constraint programming

- ❖ IBM ILOG CP
- ❖ ECLiPSe
- ❖ SCIP *

** combined with mixed-integer*

Implementation Challenges

Requirements

- ❖ Full description of functions
- ❖ Hints to algorithm
 - * convexity, search strategy

Variability

- ❖ Range of expressions recognized
 - * hence range of conversions needed
- ❖ Design of interface

The System

APIs & IDEs

- ❖ Current options
- ❖ Alternatives under consideration

AMPL in the cloud

- ❖ AMPL & solver software as a service
- ❖ Issues to be resolved

APIs (Programming Interfaces)

Current options

- ❖ AMPL scripting language
- ❖ put/get C interface
- ❖ OptiRisk Systems COM objects

Alternatives under consideration

- ❖ multiplatform C interface
- ❖ object-oriented interfaces in C++, Java, Python, . . .

Scripting Language

Programming extensions of AMPL syntax

```
for {i in WIDTHS} {
  let nPAT := nPAT + 1;
  let nbr[i,nPAT] := floor (roll_width/i);
  let {i2 in WIDTHS: i2 <> i} nbr[i2,nPAT] := 0;
};
repeat {
  solve Cutting_Opt;
  let {i in WIDTHS} price[i] := Fill[i].dual;

  solve Pattern_Gen;
  printf "\n%7.2f%11.2e  ", Number, Reduced_Cost;

  if Reduced_Cost < -0.00001 then {
    let nPAT := nPAT + 1;
    let {i in WIDTHS} nbr[i,nPAT] := Use[i];
  }
  else break;

  for {i in WIDTHS} printf "%3i", Use[i];
};
```

put/get C Interface

Send AMPL commands & receive output

- ❖ Ulong **put**(GetputInfo *g, char *s)
- ❖ int **get**(GetputInfo *g, char **kind, char **msg, Ulong *len)

Limitations

- ❖ Low-level unstructured interface
- ❖ Communication via strings

OptiRisk COM Objects

Object-oriented API

- ❖ Model management
- ❖ Data handling
- ❖ Solving

Limitations

- ❖ Windows only
- ❖ Older technology
- ❖ Built on put/get interface

API Development Directions

Multipatform C interface

- ❖ Native to AMPL code
- ❖ Similar scope to COM objects

Object-oriented interfaces

- ❖ Built on C interface

IDEs (Development Environments)

Previous & current options

- ❖ AMPL Plus
- ❖ AMPL Studio

Alternatives under consideration

- ❖ Multiplatform graphical interface
- ❖ Spreadsheet interface

AMPL Plus

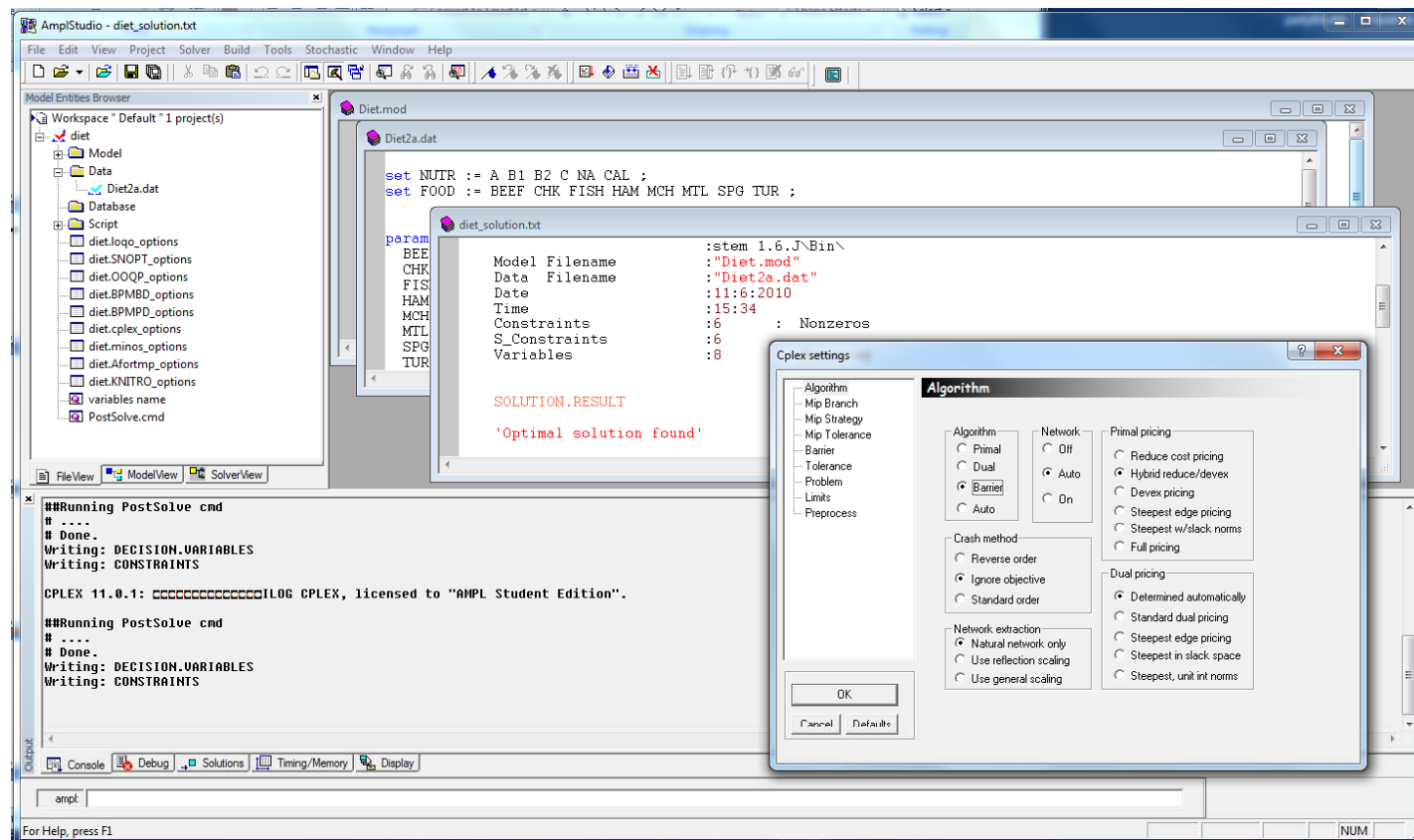
Menu-based GUI (1990s)

- ❖ Created by Compass Modeling Solutions
- ❖ Discontinued by ILOG

AMPL Studio

Menu-based GUI (2000s)

- ❖ Created by OptiRisk Systems
- ❖ Windows-based



IDE Development Directions

Multipatform graphical interface

- ❖ Focused on command-line window
 - * Same rationale as MATLAB
- ❖ Implemented using new API
- ❖ Tools for debugging, scripting, option selection . . .

Spreadsheet interface

- ❖ Data in spreadsheet tables (like Excel solver)
- ❖ AMPL model in embedded application

AMPL in the Cloud

AMPL as a service

- ❖ Solvers included
 - * optional automated solver choice
- ❖ Charges per elapsed minute
- ❖ Latest versions available

Issues to be resolved

- ❖ Licensing arrangements with solvers
- ❖ Uploading & security of data
- ❖ Limitations of cloud services